SET - 2 | GR 14 | CODE: GR14A1002

I B. Tech I Semester Regular Examinations, January, 2015
ADVANCED CALCULUS
(Common to CE, EEE, ME, ECE, CSE, BME and IT)

Time: 3 hours **Max Marks: 70**

PART – A
Answer ALL questions
All questions carry equal marks
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2 * 10 = 20 Marks

1). a If $x=\rho\cos\theta,\ y=\rho\sin\theta$, evaluate the Jacobian $\dfrac{\partial(x,y)}{\partial(\rho,\theta)}$ **[2]**

b Write the Hessian matrix for the function $f(x,y)=2x^2+3y^2-12x+18y+30$ **[2]**

c The right triangle enclosed by $x=0,\ y=0,\ x+y=3$ revolves about the $y-$axis. What is the volume of the solid so formed? **[2]**

d The spherical coordinates of a point on the sphere $x^2+y^2+z^2=9$ are $(\rho,\theta,\phi)=\left(3,\dfrac{\pi}{2},\pi\right)$. What are the corresponding Cartesian coordinates? **[2]**

e Use plane polar coordinates to evaluate $\iint_R \dfrac{dx\,dy}{\sqrt{x^2+y^2}}$ where R is the region enclosed by $x^2+y^2=1, x\ge 0,\ y\ge 0$. **[2]**

f Evaluate the triple integral $\int_0^1\int_0^2\int_0^3 (x+y+z)\,dx\,dy\,dz$ **[2]**

g Evaluate $\nabla(\ln r)$ where $\vec{r}=xi+yj+zk\ \ and\ r=|\vec{r}|$ **[2]**

h What is an irrotational field? Check mathematically if the following field is irrotational $F=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ **[2]**

i If C consists of the closed boundary $x=0,\ y=0,\ x+y=4$ what is the output of the integral $\dfrac{1}{2}\oint_C x\,dy-y\,dx$? **[2]**

j What does the operation $\iint_R \sqrt{1+{z_x}^2+{z_y}^2}\ dx\,dy$ perform on $z=5,\ x^2+y^2=1$? What is the output? **[2]**

PART – B

Answer any FIVE questions. All questions carry equal marks

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10 * 5 = 50 Marks

2. Design the largest rectangular box without a top and having a fixed surface area 300 sft. What are the dimensions and what is the maximum volume? Use Lagrange multiplier method. **[10]**

3. Give your opinion on the following solution based on changing the order of integration. If there is no error, compute the right hand side. Otherwise rectify the error and evaluate the right hand side. **[10]**

$$\int_0^2 \int_{y/2}^{y} (x+2y)\,dx\,dy \equiv \int_0^1 \int_x^{2x} (x+2y)\,dy\,dx + \int_0^2 \int_2^{x} (x+2y)\,dy\,dx$$

4. (a) Evaluate the volume of the solid generated by revolving the cycloid $x = 4(\theta - \sin\theta)$, $y = 4(1-\cos\theta)$ about the x – axis [5] **[10]**

(b) Find the arc length of the hyper cycloid $x = 2\cos^3\theta,\ y = 2\sin^3\theta$ in the parameter range $0 \le \theta \le \dfrac{\pi}{2}$ [5]

5. Evaluate the surface integral $\iint_S \vec{F}.\hat{n}\, dS$ given the vector field $\vec{F} = 18z\,i - 12\,j + 3y\,k$ and S is the surface of the plane $2x + 3y + 6z = 12$ in the first octant. **[10]**

6. Verify Green's theorem for $\oint_C \left(x + y^2\right)dx + \left(x^2 + y\right)dy$ where C is the triangle in the xy-plane with vertices $(0,0),(2,2),(0,2)$ integration being carried out counter clockwise. **[10]**

7. (a) Given the transformation equations $u = x + y + z,\ uv = y + z,\ uvw = z$, compute the Jacobian $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$ [5] **[10]**

(b) Where are the saddle points of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ located? Use Hessian Matrix to identify them.

8. (a) Evaluate by Stoke's theorem $\oint_C (2x-y)\,dx - yz^2\,dy - y^2z\,dz$ where C is the curve of intersection of $x^2 + y^2 + z^2 = 1, z = 0$ [5] **[10]**

(b) Evaluate by Gauss divergence theorem $\oiint_S \vec{F}.\hat{n}\,dS$ where $F = (2x+y)\,i + (3y+z)\,j + (4z+x)\,k$ and S is the closed hemisphere given by $x^2 + y^2 + z^2 = 16,\ z \ge 0$ [5]

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