SET - 1 | GR 14 | CODE: GR14A1001

I B. Tech I Semester Regular Examinations, January, 2015
Linear Algebra and Single Variable Calculus
(Common to CE, EEE, ME, ECE, CSE, BME and IT)

Time: 3 hours **Max Marks: 70**

PART – A
Answer ALL questions
All questions carry equal marks
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2 * 10 = 20 Marks

1). a Find *'k'* such that the matrix $A=\begin{pmatrix}1 & 2 & 1 & 2\\ 2 & 1 & 2 & 1\\ 7 & 8 & k & 8\end{pmatrix}$ is of rank 2. **[2]**

b Find the eigenvector corresponding to the largest eigenvalue of the matrix $A=\begin{pmatrix}4 & 3 & 1\\ 0 & 5 & 2\\ 0 & 0 & 8\end{pmatrix}$ **[2]**

c Find the Moore-Penrose pseudo inverse of the matrix $A=\begin{pmatrix}6 & 8\\ 3 & 4\end{pmatrix}$ **[2]**

d Find the signature of the quadratic form $Q(X)=6x_1^2-4x_1x_2+2x_2^2$ **[2]**

e Find *'c'* of the Cauchy's mean value theorem for the function pair $f(x)=\sin x$ and $g(x)=\cos x$, both defined on the interval $0\le x\le \frac{\pi}{2}$ **[2]**

f Find the exact power series expansion of the polynomial $6x^2+2x+1$ in powers of $(x-1)$ **[2]**

g If the half life of a certain radio isotope is 1200 years, find when 90% of its mass disintegrates. **[2]**

h Find an integration factor of the differential equation $y(x+y)\,dx+(x+2y-1)\,dy=0$ **[2]**

i Find the curve that passes through the point $(1,e)$ and having the property that at each point, the sub tangent is proportional to the square of the abscissa. **[2]**

j Find the *particular integral* of the differential equation $y''+4y=5\cos 2x+e^{2x}$ **[2]**

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PART – B
Answer any FIVE questions
All questions carry equal marks
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5 * 10 = 50 Marks

2. Determine the parameter λ such that the linear homogeneous system $3x_1 + 10x_2 + 5x_3 = \lambda x_1$, $-2x_1 - 3x_2 - 4x_3 = \lambda x_2$, $3x_1 + 5x_2 + 7x_3 = \lambda x_3$ has non trivial solutions. Hence solve the system for the largest real value of λ . **[10]**

3. Perform a QR factorization of the matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ by the Gram Schmidt process. **[10]**

4. (a) Approximate $\sqrt[5]{36}$ to 3 decimal places using the Lagrange's mean value theorem [4] **[10]**

(b) Find the Maclaurin's expansion of $\tan^{-1} x$ up to 3 terms [6]

5. (a) Solve the initial value problem $y' - y\cot x = 2x - x^2 \cot x, \quad y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{2} + 1$ [6] **[10]**

(b) A metallic ball with initial temperature $180^0 C$ is placed in a room with temperature $40^0 C$. After 15 minutes, the temperature of the hot body drops to $120^0 C$. Apply the Newton's law of cooling to estimate when the temperature of the body drops to $75^0 C$. [4]

6. Solve the linear differential equation $y'' - 2y' + 5y = (x^2 + 1)e^{-2x}$ **[10]**

7. (a) Prove that the eigenvalues of a skew hermitian matrix are purely imaginary or zero [4] **[10]**

(b) Find the condition number of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 5 & 5 & 6 \\ 1 & 0 & 0 \end{pmatrix}$ [6]

8. Perform a full SVD (singular value decomposition) of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. Use exact arithmetic. **[10]**

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